Code No: 181AN **R22**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year I Semester Examinations, January/February - 2024
MATRICES AND CALCULUS
(Common to EEE, CSE, IT, CSIT, CE (SE), CSE(CS), CSE(DS), CSD)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a) Define the rank of a matrix. [1]

b) Investigate the number of solutions of the system of linear equations:
$2x + 3y + 5z = 9;$
$7x + 3y - 2z = 8;$
$2x + 3y + \lambda z = \mu$; when λ =5 and μ≠9. [1]

c) Find the characteristic polynomial of $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & 3 & 9 \end{bmatrix}$. [1]

d) Is the matrix, $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$ diagonalizable? [1]

e) In the mean value theorem, $f(b) - f(a) = (b - a)f'(c)$, determine c lying between a and b, if $f(x) = x(x-1)(x-2)$, $a = 0$ and $b = 1/2$. [1]

f) Write the expression for Taylor's polynomial of n^{th} order. [1]

g) Find the value of Jacobian $\partial(u,v)/\partial(r,\theta)$ where $u = x^2 - y^2$, $v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$. [1]

h) Write the sufficient conditions for maxima and minima of functions of two variables. [1]

i) Write the relation between Cartesian coordinates (x,y,z) and cylindrical polar coordinates (e, θ, z) of a point P. [1]

j) Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum. [1]

PART - B

(50 Marks)

2. Apply Gauss Elimination method to solve the equations: [10]

$$x + 4y - z = -5$$
$$x + y - 6z = -12$$
$$3x - y - z = 4.$$

OR

3. Solve by Jacobi's iteration method, the system of linear equations (correct to 4 decimal places) [10]

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25.$$

4. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

Verify Cayley-Hamilton theorem and hence prove that:

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I = \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}.$$ [10]

OR

5.a) Find a matrix P which transform the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ to the form $P^{-1}AP = D$ where D is diagonal matrix. Hence, find A^4.

b) If $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, find A^{100} using Cayley-Hamilton Theorem. [5+5]

6.a) Find the Maclaurin series for $\sin x$.

b) Represent $f(x) = \sin x$ as Taylor series about $x = \pi/3$. [5+5]

OR

7.a) Show that $\int_0^\infty \frac{x^{m-1}}{(ax+b)^{m+n}} dx = \frac{B(m,n)}{a^m b^n}$.

b) Show that $B(m,n) = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$ for $m > 0, n > 0$. [5+5]

8.a) If $x + y + z = u, y + z = uv, z = uvw$, then find the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

b) If $u = Tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$. [5+5]

OR

9.a) If $u = xyz, v = x^2 + y^2 + z^2, w = x + y + z$, then find the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

b) If $x + y + z = 3$, find the maximum value of $\sqrt[3]{xyz}$. [5+5]

10. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$. [10]

OR

11. Find the volume of a solid bounded by the spherical surface $x^2 + y^2 + z^2 = 4a^2$ and the cylinder $x^2 + y^2 - 2ay = 0$. [10]

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